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A Stochastic Model of Urbanisation in India

Introduction

URBANIZAT1QN, i.e., the proportion of population urban, is both an antecedence and a consequence of several forces at work in a society undergoing transition. These forces include modernization, economic development; sociaf, caftural *and* political changes; and governmental policies. Even though most of the developing countries like India are at a low level of urbanization as compared to western countries, what is startling is the rapidity with which the process is taking *place* and its associated social and economic implications. So, the study of the patterns of urbanization i* of great significance and relevance to the planners and policy makers who should have a sound knowledge of the extent of urbanization that a society or a country under consideration would experience in future under certain patterns of movement towards cities,

In view of the various forces at work, the path of urbanization is not deterministic but rather stochastic in nature. The purpose of the present paper is to predict the future path of urbanization in India based on a diffusion process approximation assuming time and state space as continuous. Krishnan (1981) has applied the diffusion process approximation to study the path of human mortality and in a later study (1984) has used the same approximation to examine *the* human fertility trajectory. In both these cases, he has found the approximation to be satisfactory. Tintner and Seagupta (1972) have also observed that the log-normal approximation of the diffusion process is satisfactory for many economic characteristics.

Methodology

In order to predict the future course of urbanization in India, the log-normal approximation of the diffusion process will be employed. An excellent exposition of the process is available in Tintner and Sengupta (1972). Let $X(t)$ be the proportion of population urban at time t. These proportions have to be adjusted for definitional changes in urbanization. It is assumed here that the differentials in fertility and mortality have negligible effects on these proportions. $X(t)$ is taken as a Markovian random variable of a continuous process $[X(t), t > 0]$, in continuous time and state space $H : 0 \leqslant x \leqslant \infty$.

Let the transition probability density function be

$$f(\tau, x; t, y) = Pr\,[X(t) = y; X(\tau) = x],\ 0 < y,\ x < \infty,\ 0 \leqslant \tau < t \quad (1)$$

Consider the random variable $X(t)$ as continuous with probability unity and the transition probability density (1) satisfies the backward and forward Kolmogorov equations. Further, let $b(t, x) = b_0 x$ and $a(t, x) = a_0 x2$ denote the mean and variance respectively of the change in $X(t)$ during an infinitesimal interval Af of time, where a_0 and b_0 are independent of t.

Tintner and Dasgupta (1972) have shown that the backward and forward Kolmogorov equations are, respectively,

$$\frac{\partial f}{\partial \tau} = \frac{1}{2}\, a_0 x^2 \frac{\partial^2 f}{\partial x^2} - b_0 x \frac{\partial f}{\partial x}$$

$$\frac{\partial f}{\partial t} = \frac{1}{2}\, a_0 y^2 \frac{\partial^2 f}{\partial y^2} + (2a_0 - b_0)\, y \frac{\partial f}{\partial y} + (a_0 - b_0) f \quad (2)$$

It has been shown that the probability density function satisfying these diffusion equations (2) is the log-normal density given by

$$f(\tau, x; t, y) = \left[\frac{1}{y\{2\pi\gamma\,(t - \tau)\}^{1/2}}\right] \exp\left[\frac{-1}{2\gamma(t - \tau)} \{\ln y - \ln x - \beta\,(t - \tau)\}^2\right], \quad (3)$$

where $\gamma = a_0$ and $\beta = (b_0 - a_0/2)$

Taking '~ = 0 and assuming $Pr\,[X(0) = X_0] = 1$ as the boundary conditions, the mean and variance of $X(t)$ are found to be

$$E[X(t)] = x_0 \exp\,(b_0 t) \quad (4)$$

$$\mathrm{Var}\,[X(t)] = x_0^2 \exp\,[2b_0 t \exp\,(a_0 t - 1)] \quad (5)$$

In order to find the mean and variance of $x(t)$, we have to estimate a, and b_t. Tintner and Sengupta (1972) have employed the method of maximum likelihood to estimate the parameters ß and Y which in turn provide estimates of a_0 and b_0. Taking $x_0, x_1, \ldots, x_n$ as the observed values of $X(t)$ at time points $0, 1, \ldots, n$ the maximum likelihood estimates of ß and Y are obtained as

$$\hat{\beta} = \sum_j (\ln x_j - \ln x_{j-1})/n \tag{6}$$

$$\hat{\gamma} = \sum_j (\ln x_j - \ln x_{j-1})^2/n - \hat{\beta}^2 \qquad j = 1, 2, \ldots, n \tag{7}$$

From (6) and (7), the maximum likelihood estimates $\hat{a}_0$ and $\hat{b}_0$ are obtained as

$$\hat{a}_0 = \hat{\gamma} \tag{8}$$

$$\hat{b}_0 = \hat{\beta} + \hat{\gamma}/2 \tag{9}$$

For large samples, these estimators are shown to be the "best" estimators.

Data and Findings

In the Indian censuses urban population is polytomized into six classes.[1] Because of changing definitions adopted in the Indian censuses (see, for example, Bose, 1980) particularly from 1961, towns belonging to class VI have been excluded from the analysis. The time series on the proportion of population urban by size class of the cities and towns is shown in Table I.

Employing the log normal diffusion process approximation, the proportion urban has been projected for the years 1991, 2001, 2011 and 2021. Table 2 presents these projections for various size classes of cities and towns. It is interesting to note that the larger the cities, the faster they grow in future. However, the proportion of people living in smaller towns (class IV and class V), in general, declines with the passage of time (see Tables 1 and 2). If the present pattern of urbanization continues, the proportion of population urban living in class I cities in 1981 is expected to nearly double by the year 2021 (from 13.8 to *26* 9 per cent), whereas the proportions in class II and class III cities are expected to increase from 2.7 to 4.9 per cent and 3.3 to 4.2 per cent

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i. 'Urban population' is categorized into the following six classes by **stratifying the** cities and towns on the basis of their populations :

Class	*Population*
I	**100,000** and above
II	**50,000-99.999**
III	20,000-49,999
IV	**10,000-19,999**
V	5,000— **9,999**
VI	**Below 5,000**

TABLE l-PROPORTION *Of* POPULATION URBAN BY SIZE CLASS
OP CITIES AND TOWNS, INDIA, 1901-1981

Year	Size Class				
	/	*II*	*III*	*IV*	*V*
1901*	0.0248	0.0128	0.0179	0.0239	0.0221
1911*	0.0249	0.0112	0.0182	0.0211	0.0204
1921'	0.0283	0.0139	0.0189	0.0211	0.0212
1931*	0.0328	0.0143	0.0225	0.0228	0.0208
1941*	00491	0.0163	0.0245	0.0226	0.0213
1951+	0.0657	0.0211	0.0308	0.0260	00236
1961+	0.0800	0.0217	0.0359	0.0257	0.0144
1971#	0.1040	0.0241	00345	0-0239	0.0104
I9Sltf	0.1379	*0.0266*	0.0328	0.0217	0.0083

SOURCE:
•**Computed** from **Bose** (1980), Table 5, p. 82 and Table 7, p. 83, **and** from **United Nations (1982), Table 6, p. 30.**
fComputed from **Bose** (1980), **Table 4, p. 92 and from United Nations (1982), Table 6, p. 30.**
Computed from United Nations (1982), Table 6, p. 30 and Table **23,** p. 55.
++ **Computed from United Nations** (1982), **Table 6,** p. **30** and Table 28, p. 61.

TABLE 2-EXPECTED PROPORTION OF POPULATION URBAN BY SIZE CLASS OF CITIES AND TOWNS, 1NDIA, 1991-2021

Year	*Size Class*				
	I	*//*	***III***	*IV*	*V*
199)	0.1483	0.0348	0.0341	0.0228	0.0097
2001	0.1809	0.0389	0.0366	0.0226	0.0088
2011	0.2204	0.0435	0.0394	0.0225	0.0080
2021	0.2691	0.0486	0.0423	0.0224	0.0007

SOURCE : Projections from the log-normal fits.

respectively during the same period of time. Figure 1 shows, for example, that the expected proportions of urban people living in class I cities obtained by fitting the log-normal diffusion process approximation agree with the observed proportions, particularly between the period 1951 and 1971.

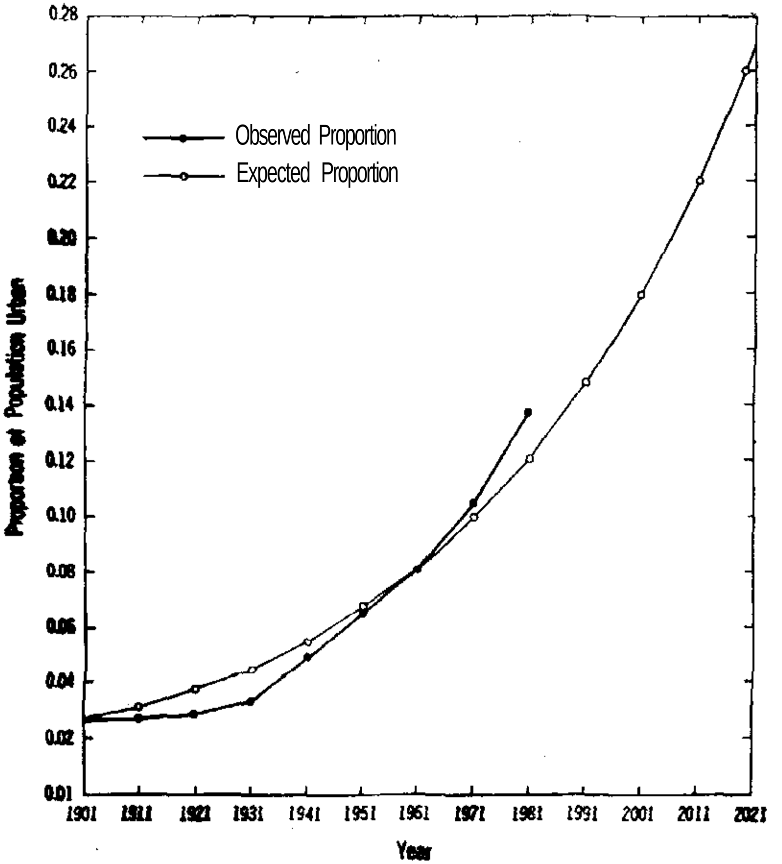

Fig. 1

Rural to urban and urban to urban migration may account for the fast growth of class I cities. Better employment opportunities resulting from concentration of industries, public offices, academic institutions, and services of public utilities in big cities encourage people to migrate both from rural areas and from small towns. On the other band, low employment potential in smaller towns with low industrial development, migration of people towards big cities, and the upgrading of smaller towns to higher classes of towns may be responsible for the slow growth of smaller towns.

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